§1. A Non-dissipative Vlasov Simulation

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In 1970's, various methods of Vlasov simulation had been extensively developed to study nonlinear and kinetic plasma phenomena in phase space. One of the most convincing methods seems to be the "splitting" scheme proposed by Cheng and Knorr[1] with the Fourier mode interpolation[2], since it keeps non-dissipative nature (or equivalently, time-reversibility) of the Vlasov-Poisson system. Time integration in the splitting scheme is performed in three steps;

$$\begin{cases} f^*(x,v) = f^n(x - v\Delta t/2, v) \\ f^{**}(x,v) = f^*(x, v - qE(x)\Delta t/m) \\ f^{n+1}(x,v) = f^{**}(x - v\Delta t/2, v) \end{cases} \quad (1)$$

The coordinate transformations, such as $x - v\Delta t/2$, are accurately calculated in Fourier space by multiplying a phase shifting factor of $\exp(-ikv\Delta t/2)$ as well as $\exp(-i\ell qE(x)\Delta t/m)$. Here, ℓ denotes a wave number in the velocity space. When $k \ll \pi/\Delta x$ and $\ell \ll \pi/\Delta v$, f is exactly preserved along the characteristics of the Vlasov equation (namely, a particle trajectory). In this report, first, we derive a generalized "splitting" scheme for the Vlasov-Poisson system with higher-oder accuracy, and then, show a non-dissipative scheme which can be applied to a drift kinetic plasma.

One may find that the splitting scheme given in Eq.(1) corresponds to the leap-frog integrator for a particle motion in an electric field $E(x)$, since f is an integral of particle motion. It is known that the leap-frog method keeping the time-reversibility is a time-centered scheme with 2nd-order accuracy. Therefore, it is straightforward to extend Eq.(1) into high-orders by using the symplectic integrator[3] for the Hamilton's equations. The generalized splitting method is, thus, obtained as

$$f^{n+1}(q,p) = \prod_{i=1}^{k} \exp(d_i \Delta t \partial V/\partial q) \exp(-c_i \Delta t \partial T/\partial p) f^n$$

$$\Rightarrow \begin{cases} f_i^*(q,p) = f_i(q - c_i \Delta t \partial T/\partial p, p) \\ f_{i+1}(q,p) = f_i^*(q, p + d_i \Delta t \partial V/\partial q) \end{cases} \quad (2)$$

for $i = 1, 2, ..., k$. Here, the Hamiltonian is separable for (q,p), that is, $H(q,p) = T(p) - V(q)$. One can choose c_i and d_i so that Eq.(2) has n-th order accuracy. For $n = 2$, the simplest set is $k = 2$, $c_1 = c_2 = 1/2$, $d_1 = 1$, and $d_2 = 0$. This leads to the splitting scheme in Eq.(1). When $n = 4$, the following set is known,

$$c_1 = c_4 = \frac{1}{2(2-2^{1/3})}, \qquad c_2 = c_3 = \frac{1-2^{1/3}}{2(2-2^{1/3})},$$
$$d_1 = d_3 = \frac{1}{2-2^{1/3}}, \quad d_2 = -\frac{2^{1/3}}{2-2^{1/3}}, \quad d_4 = 0.$$

Fig.1 shows a result of a benchmark test for 2nd- and 4th-order integrators, where the nonlinear Landau damping is simulated starting from the initial condition of $f(x,v,t=0) = F_M(v)(1 + A\cos kx)$. Used parameters are $A = 0.5$, $k = 0.5\lambda_D^{-1} = 2\pi/L$, $L = 64\Delta x$, $-10v_{th} \le v \le 10v_{th}$, $\Delta v = 10v_{th}/512$, $\Delta t = \omega_p^{-1}/8$. One finds a significant improvement of the energy conservation in Fig.1.

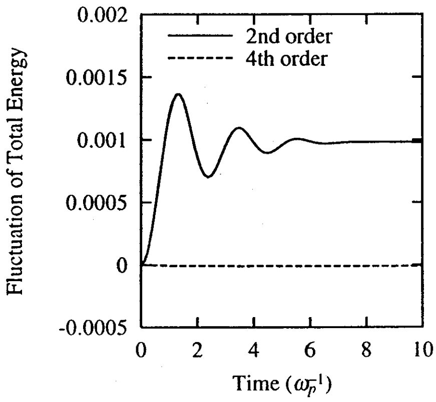

Figure 1: Comparison of 2nd- and 4th-order schemes.

Next, we have considered the drift kinetic case where the Hamiltonian of a $E \times B$ drift particle is non-separable for $\boldsymbol{x}_\perp$, that is, $H(\boldsymbol{x}, v) = mv^2/2 + e\phi(\boldsymbol{x})$. Since the explicit symplectic integrator shown above is not applicable to the non-separable Hamiltonian, we need to employ an implicit scheme. Thus, we directly integrate the drift kinetic equation using the implicit midpoint rule, $\boldsymbol{U}^{n+1} + \boldsymbol{U}^n = \Delta t \boldsymbol{F}\left([\boldsymbol{U}^{n+1} + \boldsymbol{U}^n]/2\right)$, or its 4th-order version[4]. Because both the methods are time-reversible (non-dissipative), we found a recursive growth and decay of a mode in simulations of three-mode coupling of the ion-temperature-gradient (ITG) driven instability. The newly developed code will be applied to ITG mode turbulence in near future.

Reference

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